Modeling Static and Dynamic Behavior of Routes in Mobile Ad hoc Networks

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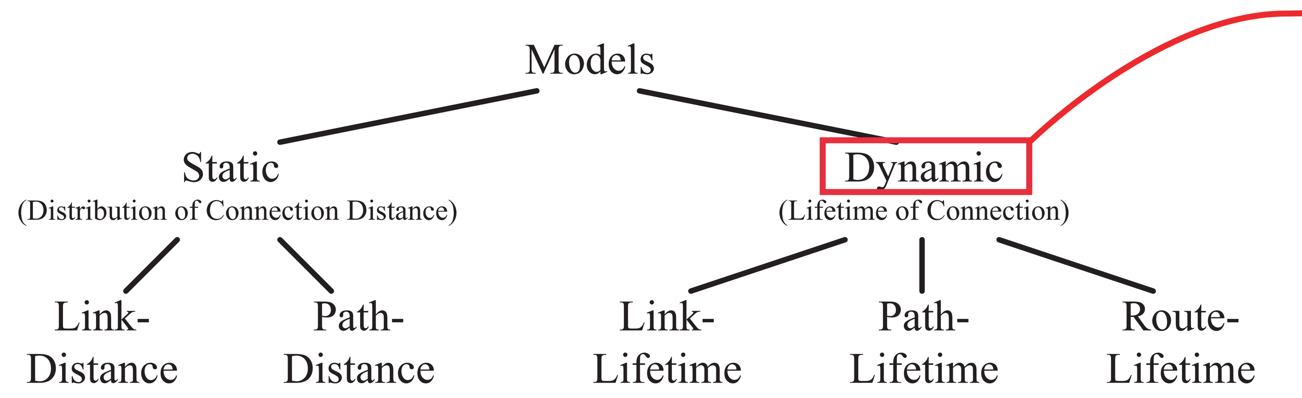
Survey and Classification of Models Describing the Static and Dynamic Behavior of Routes in Mobile Ad hoc Networks



$\theta(l_{ij})$ lifetime of link $l_{ij} = (n_i, n_j)$
$\theta(p_{ij})$ lifetime of path $p_{ij} = (n_i, ..., n_j)$
$\theta(r_{ij})$ lifetime of route $r_{ij} = \{p^1_{ij}, ..., p^n_{ij}\}$

	Object	Node Distribution	Misc. Requirements	Routing Protocol	Validation	Remark
[Samar et al. 2004]	$\theta(l_{ij})$	Nodes are uniformly distributed	Constant Node Speed		Analytical and Simulation	
[Turgut et al. 2001]	$\theta(l_{ij})$	Fixed Node Movements	GPS for Localization		Analytical	
[Tseng et al. 2003]	$\theta(p_{ij})$	Random Walk	Transmission distance in cell units	DSR	Analytical and Simulation	Cell-based model
[Bai et al. 2004]	$\theta(p_{ij})$	Freeway, Random Waypoint, Reference Point Group Mobility	High Speed Node Mobility	AODV and DSR	Analytical and Simulation	
[Gruber et al. 2003]	$\theta(p_{ij})$	Nodes are uniformly distributed	Constant Node Speed	n.a.	Analytical and Simulation	
[Nasipuri et al. 2001]	$\theta(r_{ij})$	Uniform Link-lifetime Distribution	Links are independent	DSR	Analytical and Simulation	
[Yeh et al. 2003]	$\theta(r_{ij})$			AODVM	Simulation	Reliable Nodes

Analysis and Extension of Models Describing the Static and Dynamic Behavior of Routes in Mobile Ad hoc Networks

Static Models for Distribution of Connection Distances of Links and Paths

[Miller2001] Static: *Link-Distance*, *Path-Distance* [Hollick2004]

Static: *Link-Distance*, *Path-Distance* [Hollick2004] (extended)

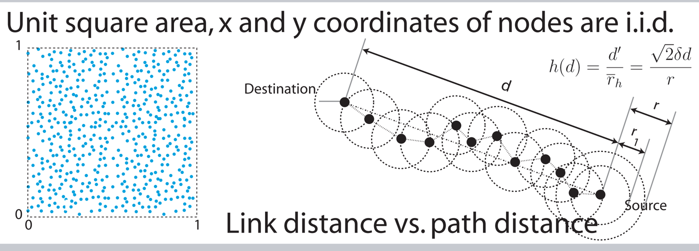
Dynamic Models for Lifetime of Links, Paths, and Routes

Dynamic: *Link-Lifetime*, *Path-Lifetime*, *Route-Lifetime* [Nasipuri et al. 2001]

Dynamic: *Link-Lifetime*, *Path-Lifetime*, *Route-Lifetime* [Nasipuri et al. 2001]

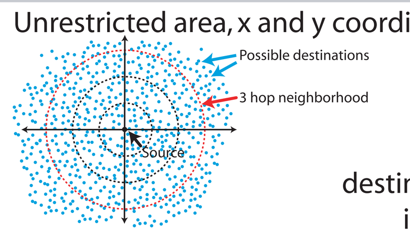
Basics

Unit square area, x and y coordinates of nodes are i.i.d.



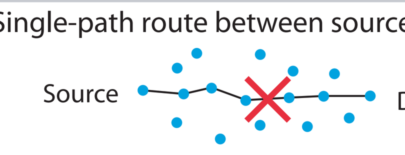
Link distance vs. path distance

Unrestricted area, x and y coordinates of nodes are i.i.d.



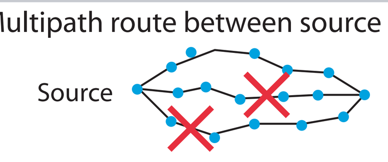
Number of possible destinations for each source increases with distance

Single-path route between source and destination



Link breaks (uniformly distributed) lead to route breaks (exponentially distributed)

Multipath route between source and destination



Link breaks lead to path breaks and, depending on the number of multipaths, to route breaks

Model

PDF of distance between two randomly selected nodes

$$f_d(x) = \begin{cases} 2x\,(x^2 - 4x + \pi) & , x \le 1 \\ 8x\sqrt{x^2-1} - 2x^3 - 4x \\ \quad + 4x\left(arcsin(\frac{1}{x}) - arccos(\frac{1}{x})\right) & , 1 < x \le \sqrt{2} \end{cases}$$

Probability measure for succesfully established routes

$$f^m_d(x) = \begin{cases} (1-q)^{h(x)}\,2x\,(x^2 - 4x + \pi) & , x \le 1 \\ (1-q)^{h(x)}\,(8x\sqrt{x^2-1} - 2x^3 - 4x) \\ \quad + (1-q)^{h(x)}\,4x\left(arcsin(\frac{1}{x}) - arccos(\frac{1}{x})\right) & , 1 < x \le \sqrt{2} \end{cases}$$

Number of destinations in distance *d*

$$g(h(d)) - g(h(d)-1) = \begin{cases} \frac{\rho\pi r^2}{2\delta^2}\,(2h(d)-1) & , h \ge 1 \\ \frac{\rho\pi r^2}{2\delta^2}\,h(d)^2 & , 0 \le h < 1 \\ 0 & , otherwise \end{cases}$$

Measure for the number of succesfully established routes

$$f^m_d(x) = \begin{cases} (1-q)^{h(x)}\,(2h(x)-1)\,\frac{\rho\pi r^2}{2\delta^2} & , x \ge \frac{r}{\sqrt{2}\delta} \\ (1-q)^{h(x)}\,h(x)^2\,\frac{\rho\pi r^2}{2\delta^2} & , 0 \le x < \frac{r}{\sqrt{2}\delta} \end{cases}$$

PDF of route lifetime for multipath routes (includes the special case of single-path routes)

$$f_{\theta(r_{ij})}(t) = \sum_{k=1}^{n}\left(\lambda(p^k_{ij})\,e^{-\lambda(p^k_{ij})\,t}\prod_{\substack{l=1\\l\ne k}}^{n}\left(1 - e^{-\lambda(p^l_{ij})\,t}\right)\right)$$

$$\lambda(p^k_{ij}) = \frac{\eta_k}{\mu(p^k_{ij})} = \frac{|p^k_{ij}|}{\mu(p^k_{ij})}$$

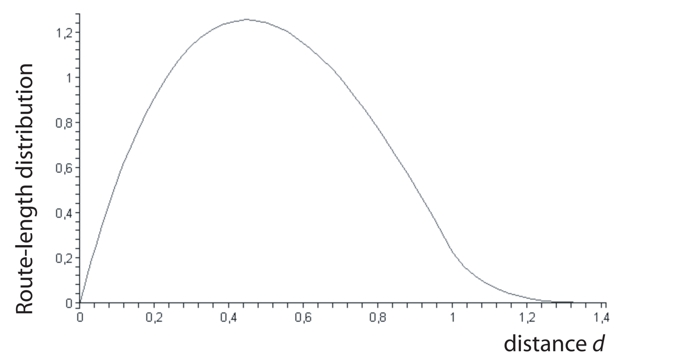
if all paths link-disjoint (only source node maintains alternate routes)

$$f_{\theta(r_{ij})}(t) = \sum_{k=1}^{\eta}\left(f_{\theta(w_k)}(t)\prod_{\substack{l=1\\l\ne k}}^{\eta}\left(1 - F_{\theta(w_l)}(t)\right)\right)$$

if alternate paths are not link-disjoint, but each node on the primary route maintains an alternate (backup) route

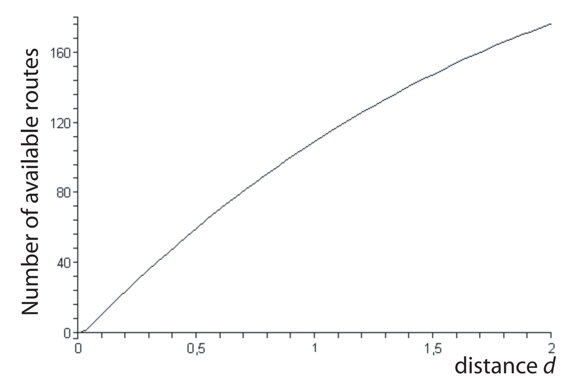
Results

PDF of distance between two randomly selected nodes



The shown results borrow the parameter set (radio range, node density, etc.) from [Hollick2004]

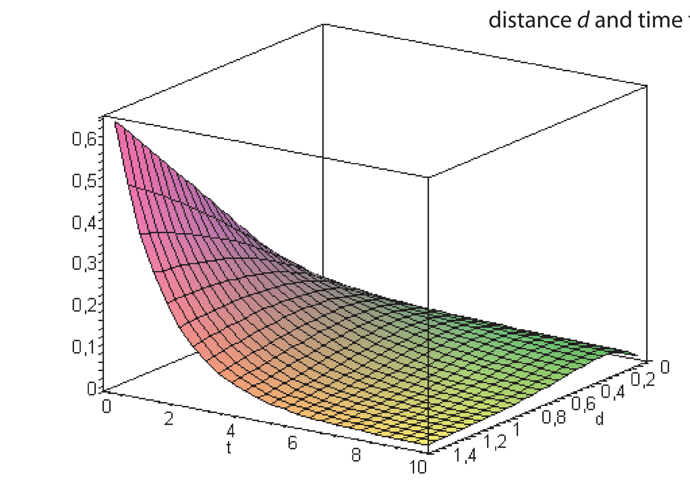
Number of possibly established routes of distance *d*



The shown results borrow the parameter set (radio range, node density, etc.) from [Hollick2004]

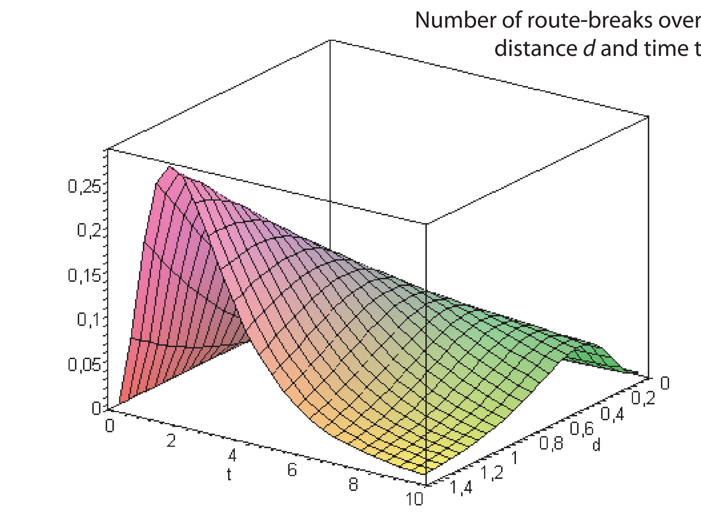
Model prediction for n=1 paths (single-path)

Number of route-breaks over distance *d* and time t



Model prediction for n=3 equal-length multipaths

Number of route-breaks over distance *d* and time t



Formulation of a Combined (Static & Dynamic) Model

Number of available routes of length *d* at time θ

$$f^m_{d,\theta}(x) = f^m_d(x) - \int_0^{\theta} f_{d,\theta}(x,t)\,dt$$

Routes at time θ; Broken routes until time θ; All routes at time θ=0

Number of route breaks of length *d* until time θ for the generalized multipath case

$$f_{d,\theta}(x,t) = \begin{cases} (1-q)^{h(x)}\,2x\,(x^2-4x+\pi) \\ \quad \left(\sum_{i=1}^{g_\rho(d)}\left(2q\,h_i(x)\,e^{-2qt\,h_i(x)}\prod_{\substack{j=1\\j\ne i}}^{g_\rho(d)}\left(1 - e^{-2qt\,h_j(x)}\right)\right)\right) & , x \le 1 \\ \left((1-q)^{h(x)}\,(8x\sqrt{x^2-1} - 2x^3 - 4x)\right. \\ \quad \left. + (1-q)^{h(x)}\,4x\left(arcsin(\frac{1}{x}) - arccos(\frac{1}{x})\right)\right) \\ \quad \left(\sum_{i=1}^{g_\rho(d)}\left(2q\,h_i(x)\,e^{-2qt\,h_i(x)}\prod_{\substack{j=1\\j\ne i}}^{g_\rho(d)}\left(1 - e^{-2qt\,h_j(x)}\right)\right)\right) & , 1 < x \le \sqrt{2} \end{cases}$$

Number of route breaks of length *d* until time θ for the special case of equal-length multipaths

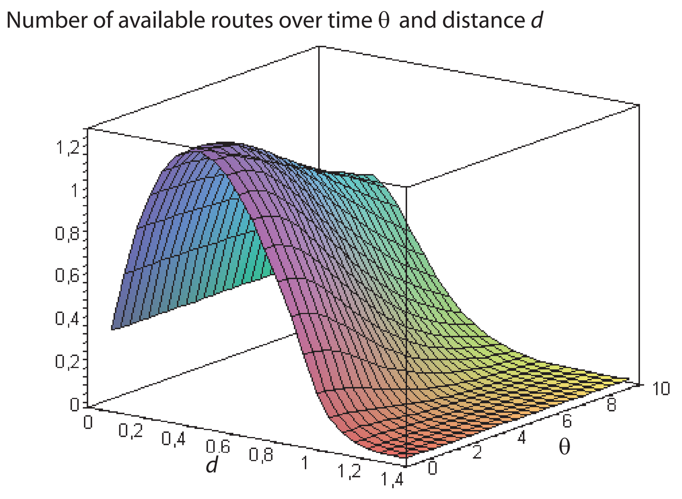
$$\int_0^{\theta} f_{d,\theta}(x,t)\,dt = \begin{cases} (1-q)^{h(x)}\,2x\,(x^2-4x+\pi)\left(1 - e^{-2q\theta\,h(x)}\right)^n & , x \le 1 \\ \left((1-q)^{h(x)}\,(8x\sqrt{x^2-1} - 2x^3 - 4x)\right. \\ \quad \left. + (1-q)^{h(x)}\,4x\left(arcsin(\frac{1}{x}) - arccos(\frac{1}{x})\right)\right) \\ \quad \left(1 - e^{-2q\theta\,h(x)}\right)^n & , 1 < x \le \sqrt{2} \end{cases}$$

Number of available routes of length *d* at time θ

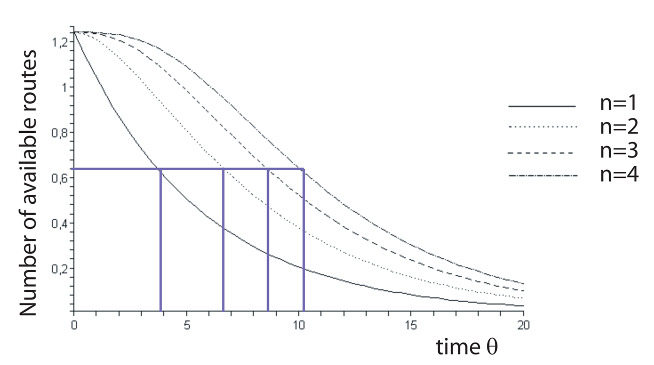
$$f^m_{d,\theta}(x) = \begin{cases} (1-q)^{h(x)}\,2x\,(x^2-4x+\pi) \\ \quad \left(1 - \left(1 - e^{-2q\theta\,h(x)}\right)^n\right) & , x \le 1 \\ \left((1-q)^{h(x)}\,(8x\sqrt{x^2-1} - 2x^3 - 4x)\right. \\ \quad \left. + (1-q)^{h(x)}\,4x\left(arcsin(\frac{1}{x}) - arccos(\frac{1}{x})\right)\right) \\ \quad \left(1 - \left(1 - e^{-2q\theta\,h(x)}\right)^n\right) & , 1 < x \le \sqrt{2} \end{cases}$$

Model predictions combining [Hollick2004] with [Nasipuri et al. 2001]

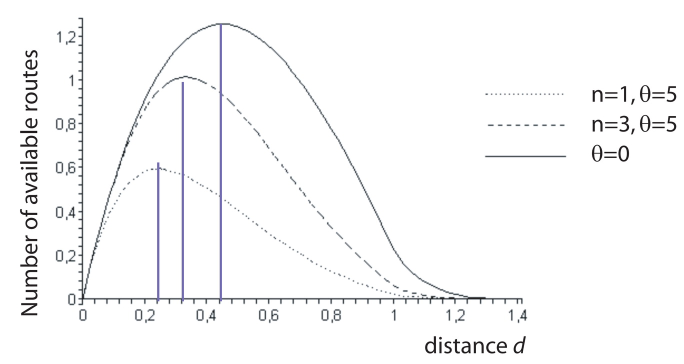
Number of available routes over time θ and distance *d*



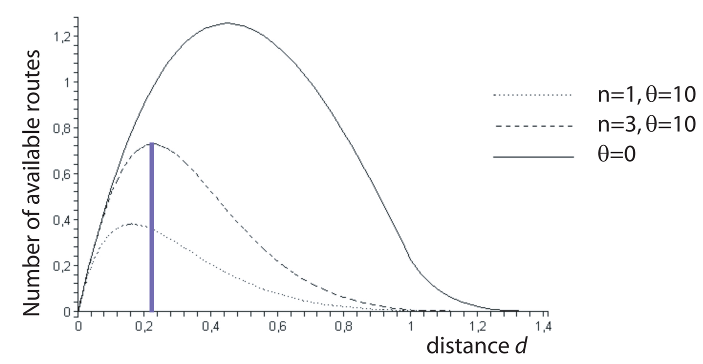
Number of available routes over time θ for selected numbers of equal-length multipaths and distance *d*=0.4



Number of available routes over distance *d* for selected numbers of equal-length multipaths and time θ=0 and θ=5 units (the number of available routes is equal for n=1, n=3 in case θ=0)



Number of available routes over distance *d* for selected numbers of equal-length multipaths and time θ=0 and θ=10 units (the number of available routes is equal for n=1, n=3 in case θ=0)



Application of the Model: Analysis of Strategies for Efficient Usage of Multipath Routes

Lifetime of Multipath Routes

We analyze the lifetime of multipath routes., i.e., the time until the route is no longer usable for data transport.

Let us assume a uniformly distributed (μ) link lifetime. → Path lifetime is exponentially distributed $F_{\theta(p^k_{ij})}(t) = 1 - e^{\frac{\eta_k}{\mu}t}$ with $\eta_k = |p^k_{ij}|$

→ Route lifetime is $f_{\theta(r_{ij})}(t) = \sum_{k=1}^{n}\left(\lambda(p^k_{ij})\,e^{-\lambda(p^k_{ij})\,t}\prod_{\substack{l=1\\l\ne k}}^{n}\left(1 - e^{-\lambda(p^l_{ij})\,t}\right)\right)$

The expected route lifetime can now be calculated for the case of n=1,2,... multipaths to be:

$$E(\theta(r_{ij})) = \begin{cases} \frac{\mu}{\eta} & , n = 1 \\ \mu\frac{\eta_1^2 + \eta_1\eta_2 + \eta_2^2}{\eta_1(\eta_1+\eta_2)\eta_2} & , n = 2 \end{cases}$$

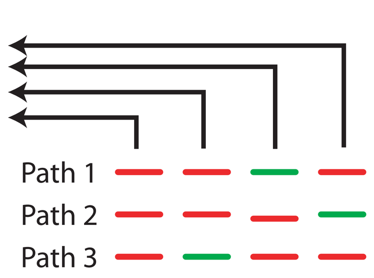
Example:

CDF of a route break in a 3 path system with 2/3 FEC:

$$\begin{aligned} F_{\theta(r_{ij})}(t) &= F_{\theta(p_1)}(t)\,F_{\theta(p_2)}(t)\,F_{\theta(p_3)}(t) \\ &+ F_{\theta(p_1)}(t)\,F_{\theta(p_2)}(t)\,(1 - F_{\theta(p_3)}(t)) \\ &+ F_{\theta(p_1)}(t)\,(1 - F_{\theta(p_2)}(t))\,F_{\theta(p_3)}(t) \\ &+ (1 - F_{\theta(p_1)}(t))\,F_{\theta(p_2)}(t)\,F_{\theta(p_3)}(t) \end{aligned}$$

For equal path lengths: $F_{\theta(r_{ij})}(t) = 1 - 3e^{-\frac{2\eta}{\mu}t} + 2e^{-\frac{3\eta}{\mu}t}$

Expected value: $E(\theta(r_{ij})) = \frac{5\mu}{6\eta}$



Transport Capacity of Multipath Routes

We analyze the transport capacity of multipath routes , i.e., the amount of traffic that can be transferred until the established routes fail. In particular, we investigate naive and advanced strategies for usage of the individual paths.

	Singe-path	2 paths, independent	3 paths, independent	3 paths, 2/3 FEC	3 paths, 2/3 FEC until route break, no FEC after route break
Expected value of the route lifetime	$\frac{\mu}{\eta}$	$\frac{3\mu}{2\eta}$	$\frac{11\mu}{6\eta}$	$\frac{5\mu}{6\eta}$	$\frac{11\mu}{6\eta}$
Transmission rate (relative to single-path)	1	1	1	2	$\frac{16}{11}$
Transmission capacity during the lifetime of the route	1	$\frac{3}{2} = 1.5$	$\frac{11}{6} \approx 1.83$	$\frac{10}{6} \approx 1.67$	$\frac{16}{6} \approx 2.67$

Summary

We developed a unified model to describe both, dynamic as well as static aspects of routes in ad hoc networks:
- Static aspects are covered using the distribution of link- and path-distances.
- Dynamic aspects are covered using the lifetime of links, paths, and routes.

We have shown the applicability of our model for the evaluation of efficient ad hoc communication:
- The lifetime of multipath routes has been analysed for different numbers of multipath.
- The transport capacity of the aforementioned multipath routes has been analysed for various path-usage strategies.

Nomenclature

$N = \{n_1, ..., n_m\}$	set of nodes n_i
$L = \{l_1, ..., l_m\}$	set of links with link $l_{ij} = (n_i, n_j)$
$P = \{p_1, ..., p_m\}$	set of paths with path $p_{ij} = (n_i, ..., n_j)$
$R = \{r_1, ..., r_n\}$	set of routes with route $r_{ij} = \{p^1_{ij}, ..., p^n_{ij}\}$
d_{ij}	euclidean distance between n_i and n_j
q_{ij}	probability for failure of link $l_{ij} = (n_i, n_j)$
$\overline{r}_h$	average routing progress per hop
r	wireless transmission range
ρ	density of nodes
$g(h)$	number of nodes reachable in h hops
$h_{ij}(d)$	number of hops in path p_{ij} of distance d
n	number of paths in route r_{ij} (i.e., $n = \lvert r_{ij}\rvert$)
η_k	length of path k in route r_{ij} (i.e., $\eta_k = \lvert p^k_{ij}\rvert$)
η	length of longest path in route r_{ij} (i.e., $\eta = max(\lvert p^1_{ij}\rvert, ..., \lvert p^n_{ij}\rvert)$)
$\mu(p^k_{ij})$	expected path lifetime for path $p^k_{ij} = (n_i, ..., n_j)$
$F_{d,\theta}(x,t)$	cumulative distribution function over distance d and time θ
$f_{d,\theta}(x,t)$	probability density function over distance d and time θ
$f^m_{d,\theta}(x,t)$	probability measure function over distance d and time θ